Методы оптимизации.

Отчет по лабораторной работе №2

Работа выполнена группой:

Дзюба Мария M3235  
Карасева Екатерина M3235  
Рындина Валерия M3235

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Цель работы: Изучить и реализовать градиентные методы, провести анализ их работы и сравнение.

* 1. Постановка задачи:  
     Реализовать алгоритмы:

метод градиентного спуска;

метод наискорейшего спуска;

метод сопряженных градиентов.

Оцените, как меняется скорость сходимости, если для поиска величины шага использовать различные методы одномерного поиска.

* 1. Решение задачи:
     + Вычислительная схема методов:  
       f(x) дифференцируема в En, xk+1 = xk + αkpk, k ∈ N, где pk определяется с учетом информации о частных производных, величина αk > 0 такова, что: f(xk+1) < f(xk).  
       Остановка итерационного процесса: ║∇f(xk)║< ε
     + Метод градиентного спуска:
       1. Вычислительная схема данного метода:  
          Предполагаем, что pk = -∇f(xk), тогда если ∇f(xk) ≠ 0, то (∇f(xk), pk) < 0, и, следовательно, pk – направление убывания f(x), таким образом, найдутся такие ak > 0, что выполнится условие: f(xk+1) < f(xk)
       2. Задача минимизации:   
          f(x1, x2) = 20\*(x1)2 + (x2)2 – 7\*x1 + 3\*x2 + 2  
          a = 1.0  
          ε = 0.001
       3. Численный результат решения:  
          минимум функции: -0,862481  
          вектор минимума: [0,175976, -1,500425]
       4. Вектор поиска решения в виде таблицы:

|  |  |  |
| --- | --- | --- |
| № | Вектор минимума | Значение минимума |
| 0 | [0,000000, 0,000000] | 2,000000 |
| 1 | [0,229786, -0,098480] | 1,161789 |
| 2 | [0,075807, -0,295432] | 0,785270 |
| 3 | [0,289500, -0,425183] | 0,554936 |
| 4 | [0,063188, -0,531403] | 0,325720 |
| 5 | [0,292593, -0,630767] | 0,169629 |
| 6 | [0,058097, -0,717435] | 0,023236 |
| 7 | [0,295170, -0,796785] | -0,079173 |
| 8 | [0,055229, -0,866990] | -0,174898 |
| 9 | [0,296932, -0,930862] | -0,241231 |
| 10 | [0,053475, -0,987681] | -0,304661 |
| 11 | [0,298099, -1,039244] | -0,347135 |
| 12 | [0,052366, -1,085233] | -0,389685 |
| 13 | [0,298866, -1,126918] | -0,416455 |
| 14 | [0,051654, -1,164148] | -0,445416 |
| 15 | [0,299368, -1,197872] | -0,461869 |
| 16 | [0,051193, -1,228017] | -0,481959 |
| 17 | [0,299698, -1,255313] | -0,491637 |
| 18 | [0,050893, -1,279724] | -0,505925 |
| 19 | [0,299914, -1,301823] | -0,511156 |
| 20 | [0,050697, -1,321592] | -0,521645 |
| 21 | [0,300055, -1,339487] | -0,523958 |
| 22 | [0,050569, -1,355498] | -0,531956 |
| 23 | [0,300148, -1,369990] | -0,532355 |
| 24 | [0,050485, -1,382958] | -0,538721 |
| 25 | [0,175347, -1,388827] | -0,850138 |
| 26 | [0,167558, -1,513584] | -0,861208 |
| 27 | [0,175338, -1,512874] | -0,862332 |
| 28 | [0,173523, -1,509415] | -0,862368 |
| 29 | [0,175384, -1,508822] | -0,862419 |
| 30 | [0,174102, -1,507348] | -0,862430 |
| 31 | [0,175910, -1,506608] | -0,862440 |
| 32 | [0,174074, -1,505942] | -0,862448 |
| 33 | [0,175934, -1,505345] | -0,862454 |
| 34 | [0,174056, -1,504808] | -0,862459 |
| 35 | [0,175949, -1,504326] | -0,862463 |
| 36 | [0,174044, -1,503891] | -0,862467 |
| 37 | [0,175958, -1,503502] | -0,862469 |
| 38 | [0,174037, -1,503151] | -0,862472 |
| 39 | [0,175964, -1,502835] | -0,862473 |
| 40 | [0,174032, -1,502551] | -0,862475 |
| 41 | [0,175968, -1,502296] | -0,862476 |
| 42 | [0,174029, -1,502066] | -0,862477 |
| 43 | [0,175971, -1,501860] | -0,862478 |
| 44 | [0,174027, -1,501673] | -0,862478 |
| 45 | [0,175973, -1,501506] | -0,862479 |
| 46 | [0,174025, -1,501355] | -0,862479 |
| 47 | [0,175974, -1,501220] | -0,862480 |
| 48 | [0,174025, -1,501098] | -0,862480 |
| 49 | [0,175975, -1,500988] | -0,862480 |
| 50 | [0,174024, -1,500889] | -0,862480 |
| 51 | [0,175975, -1,500800] | -0,862480 |
| 52 | [0,174024, -1,500720] | -0,862480 |
| 53 | [0,175975, -1,500648] | -0,862481 |
| 54 | [0,174023, -1,500583] | -0,862481 |
| 55 | [0,175976, -1,500525] | -0,862481 |
| 56 | [0,174023, -1,500473] | -0,862481 |
| 57 | [0,175976, -1,500425] | -0,862481 |

* + - Метод наискорейшего спуска:
      1. Вычислительная схема данного метода:  
         pk = -∇f(xk), ak – находится из решения задачи одномерной минимизации:  
         Фk(a) -> min, Фk(a) = f(xk – a\*∇f(xk)), a > 0
      2. Задача минимизации:   
         f(x1, x2) = 20\*(x1)2 + (x2)2 – 7\*x1 + 3\*x2 + 2  
         ε = 0.001
      3. Численный результат решения:  
         минимум функции: -0,862500  
         вектор минимума: [0,174947, -1,499543]
      4. Вектор поиска решения в виде таблицы:

|  |  |  |
| --- | --- | --- |
| № | Вектор минимума | Значение минимума |
| 0 | [0,000000, 0,000000] | 2,000000 |
| 1 | [0,205291, -0,087982] | 1,149646 |
| 2 | [0,052069, -0,445102] | 0,552552 |
| 3 | [0,196308, -0,506989] | 0,132651 |
| 4 | [0,088561, -0,758055] | -0,162584 |
| 5 | [0,189983, -0,801582] | -0,370223 |
| 6 | [0,114196, -0,978222] | -0,516306 |
| 7 | [0,185539, -1,008833] | -0,619034 |
| 8 | [0,132243, -1,133022] | -0,691265 |
| 9 | [0,182411, -1,154551] | -0,742067 |
| 10 | [0,144926, -1,241914] | -0,777803 |
| 11 | [0,180213, -1,257055] | -0,802934 |
| 12 | [0,153851, -1,318487] | -0,820607 |
| 13 | [0,178666, -1,329136] | -0,833037 |
| 14 | [0,160125, -1,372344] | -0,841779 |
| 15 | [0,177578, -1,379834] | -0,847927 |
| 16 | [0,164539, -1,410219] | -0,852251 |
| 17 | [0,176813, -1,415486] | -0,855292 |
| 18 | [0,167643, -1,436859] | -0,857431 |
| 19 | [0,176275, -1,440563] | -0,858935 |
| 20 | [0,169826, -1,455592] | -0,859993 |
| 21 | [0,175897, -1,458198] | -0,860736 |
| 22 | [0,171361, -1,468769] | -0,861260 |
| 23 | [0,175631, -1,470601] | -0,861628 |
| 24 | [0,172441, -1,478035] | -0,861887 |
| 25 | [0,175444, -1,479323] | -0,862069 |
| 26 | [0,173200, -1,484553] | -0,862197 |
| 27 | [0,175312, -1,485459] | -0,862287 |
| 28 | [0,173734, -1,489135] | -0,862350 |
| 29 | [0,175219, -1,489773] | -0,862394 |
| 30 | [0,174110, -1,492359] | -0,862426 |
| 31 | [0,175154, -1,492808] | -0,862448 |
| 32 | [0,174374, -1,494626] | -0,862463 |
| 33 | [0,175109, -1,494941] | -0,862474 |
| 34 | [0,174560, -1,496221] | -0,862482 |
| 35 | [0,175076, -1,496443] | -0,862487 |
| 36 | [0,174690, -1,497342] | -0,862491 |
| 37 | [0,175054, -1,497498] | -0,862494 |
| 38 | [0,174782, -1,498131] | -0,862496 |
| 39 | [0,175038, -1,498240] | -0,862497 |
| 40 | [0,174847, -1,498685] | -0,862498 |
| 41 | [0,175027, -1,498762] | -0,862498 |
| 42 | [0,174892, -1,499075] | -0,862499 |
| 43 | [0,175019, -1,499130] | -0,862499 |
| 44 | [0,174924, -1,499350] | -0,862499 |
| 45 | [0,175013, -1,499388] | -0,862500 |
| 46 | [0,174947, -1,499543] | -0,862500 |

* + - Метод сопряженных градиентов.
      1. Вычислительная схема данного метода:  
         p0 = = -∇f(x0), x0 ∈ En  
         для квадратичной функции:  
         ak = ;  
         pk+1 = -∇f(xk+1) + bkpk;  
         bk =
      2. Задача минимизации:   
         f(x1, x2) = 20\*(x1)2 + (x2)2 – 7\*x1 + 3\*x2 + 2  
         ε = 0.001
      3. Численный результат решения:  
         минимум функции: -0,862481  
         вектор минимума: [0,175976, -1,500425]
      4. Вектор поиска решения в виде таблицы:

|  |  |  |
| --- | --- | --- |
| № | Вектор минимума | Значение минимума |
| 0 | [0,000000, 0,000000] | 2,000000 |
| 1 | [0,205258, -0,087968] | 1,149646 |
| 2 | [0,307887, -0,131951] | 1,362235 |
| 3 | [0,141988, -0,217346] | 0,804496 |
| 4 | [0,059039, -0,260044] | 0,943930 |
| 5 | [0,206046, -0,338640] | 0,505533 |
| 6 | [0,279549, -0,377938] | 0,615133 |
| 7 | [0,146805, -0,449171] | 0,257640 |
| 8 | [0,080433, -0,484788] | 0,347013 |
| 9 | [0,200517, -0,549245] | 0,054457 |
| 10 | [0,260559, -0,581474] | 0,127596 |
| 11 | [0,151913, -0,639793] | -0,111883 |
| 12 | [0,097590, -0,668952] | -0,052013 |
| 13 | [0,195888, -0,721717] | -0,248049 |
| 14 | [0,245038, -0,748100] | -0,199040 |
| 15 | [0,156101, -0,795839] | -0,359514 |
| 16 | [0,111633, -0,819709] | -0,319396 |
| 17 | [0,192099, -0,862902] | -0,450759 |
| 18 | [0,232332, -0,884499] | -0,417918 |
| 19 | [0,159529, -0,923578] | -0,525451 |
| 20 | [0,123128, -0,943118] | -0,498568 |
| 21 | [0,188997, -0,978476] | -0,586594 |
| 22 | [0,221932, -0,996154] | -0,564587 |
| 23 | [0,162336, -1,028145] | -0,636645 |
| 24 | [0,132538, -1,044140] | -0,618630 |
| 25 | [0,186458, -1,073083] | -0,677616 |
| 26 | [0,213418, -1,087555] | -0,662870 |
| 27 | [0,164633, -1,113742] | -0,711155 |
| 28 | [0,140241, -1,126835] | -0,699084 |
| 29 | [0,184380, -1,150528] | -0,738610 |
| 30 | [0,206449, -1,162375] | -0,728729 |
| 31 | [0,166514, -1,183811] | -0,761084 |
| 32 | [0,146546, -1,194530] | -0,752995 |
| 33 | [0,182678, -1,213925] | -0,779482 |
| 34 | [0,200744, -1,223622] | -0,772860 |
| 35 | [0,168053, -1,241170] | -0,794542 |
| 36 | [0,151708, -1,249944] | -0,789121 |
| 37 | [0,181285, -1,265820] | -0,806870 |
| 38 | [0,196074, -1,273759] | -0,802433 |
| 39 | [0,169313, -1,288123] | -0,816961 |
| 40 | [0,155933, -1,295305] | -0,813329 |
| 41 | [0,180145, -1,308302] | -0,825222 |
| 42 | [0,192251, -1,314800] | -0,822249 |
| 43 | [0,170345, -1,326559] | -0,831985 |
| 44 | [0,159392, -1,332438] | -0,829551 |
| 45 | [0,179212, -1,343077] | -0,837520 |
| 46 | [0,189121, -1,348397] | -0,835528 |
| 47 | [0,171189, -1,358022] | -0,842052 |
| 48 | [0,162223, -1,362835] | -0,840421 |
| 49 | [0,178448, -1,371544] | -0,845761 |
| 50 | [0,186560, -1,375898] | -0,844426 |
| 51 | [0,171881, -1,383778] | -0,848798 |
| 52 | [0,164541, -1,387717] | -0,847705 |
| 53 | [0,177822, -1,394847] | -0,851283 |
| 54 | [0,184463, -1,398411] | -0,850389 |
| 55 | [0,172447, -1,404861] | -0,853318 |
| 56 | [0,166438, -1,408086] | -0,852586 |
| 57 | [0,177310, -1,413922] | -0,854984 |
| 58 | [0,182746, -1,416840] | -0,854384 |
| 59 | [0,172910, -1,422120] | -0,856347 |
| 60 | [0,167992, -1,424760] | -0,855857 |
| 61 | [0,176891, -1,429537] | -0,857463 |
| 62 | [0,181341, -1,431926] | -0,857062 |
| 63 | [0,173289, -1,436248] | -0,858377 |
| 64 | [0,169263, -1,438409] | -0,858048 |
| 65 | [0,176548, -1,442319] | -0,859125 |
| 66 | [0,180191, -1,444275] | -0,858856 |
| 67 | [0,173599, -1,447813] | -0,859737 |
| 68 | [0,170304, -1,449582] | -0,859517 |
| 69 | [0,176267, -1,452783] | -0,860238 |
| 70 | [0,179249, -1,454384] | -0,860058 |
| 71 | [0,173853, -1,457280] | -0,860649 |
| 72 | [0,171156, -1,458728] | -0,860501 |
| 73 | [0,176037, -1,461348] | -0,860985 |
| 74 | [0,178478, -1,462659] | -0,860864 |
| 75 | [0,174061, -1,465030] | -0,861259 |
| 76 | [0,171853, -1,466215] | -0,861161 |
| 77 | [0,175849, -1,468360] | -0,861484 |
| 78 | [0,177847, -1,469433] | -0,861403 |
| 79 | [0,174232, -1,471373] | -0,861669 |
| 80 | [0,172424, -1,472344] | -0,861602 |
| 81 | [0,175695, -1,474100] | -0,861820 |
| 82 | [0,177331, -1,474978] | -0,861765 |
| 83 | [0,174371, -1,476566] | -0,861943 |
| 84 | [0,172891, -1,477361] | -0,861899 |
| 85 | [0,175569, -1,478798] | -0,862044 |
| 86 | [0,176908, -1,479517] | -0,862008 |
| 87 | [0,174485, -1,480817] | -0,862127 |
| 88 | [0,173274, -1,481468] | -0,862097 |
| 89 | [0,175466, -1,482644] | -0,862194 |
| 90 | [0,176562, -1,483233] | -0,862170 |
| 91 | [0,174579, -1,484297] | -0,862250 |
| 92 | [0,173587, -1,484830] | -0,862230 |
| 93 | [0,175381, -1,485793] | -0,862295 |
| 94 | [0,176279, -1,486274] | -0,862279 |
| 95 | [0,174655, -1,487146] | -0,862332 |
| 96 | [0,173843, -1,487582] | -0,862319 |
| 97 | [0,175312, -1,488370] | -0,862363 |
| 98 | [0,176047, -1,488764] | -0,862352 |
| 99 | [0,174718, -1,489478] | -0,862388 |
| 100 | [0,174053, -1,489834] | -0,862379 |
| 101 | [0,175256, -1,490480] | -0,862408 |
| 102 | [0,175857, -1,490803] | -0,862401 |
| 103 | [0,174769, -1,491386] | -0,862425 |
| 104 | [0,174225, -1,491678] | -0,862419 |
| 105 | [0,175209, -1,492207] | -0,862438 |
| 106 | [0,175701, -1,492471] | -0,862433 |
| 107 | [0,174811, -1,492949] | -0,862450 |
| 108 | [0,174365, -1,493188] | -0,862446 |
| 109 | [0,175171, -1,493621] | -0,862459 |
| 110 | [0,175574, -1,493837] | -0,862455 |
| 111 | [0,174845, -1,494228] | -0,862466 |
| 112 | [0,174481, -1,494424] | -0,862464 |
| 113 | [0,175140, -1,494778] | -0,862472 |
| 114 | [0,175470, -1,494955] | -0,862470 |
| 115 | [0,174873, -1,495275] | -0,862477 |
| 116 | [0,174575, -1,495435] | -0,862476 |
| 117 | [0,175115, -1,495725] | -0,862481 |
| 118 | [0,175385, -1,495870] | -0,862480 |
| 119 | [0,174896, -1,496132] | -0,862485 |
| 120 | [0,174652, -1,496263] | -0,862484 |
| 121 | [0,175094, -1,496501] | -0,862488 |
| 122 | [0,175315, -1,496619] | -0,862487 |
| 123 | [0,174915, -1,496834] | -0,862490 |
| 124 | [0,174715, -1,496941] | -0,862489 |
| 125 | [0,175077, -1,497135] | -0,862492 |
| 126 | [0,175258, -1,497233] | -0,862491 |
| 127 | [0,174930, -1,497408] | -0,862493 |
| 128 | [0,174767, -1,497496] | -0,862493 |
| 129 | [0,175063, -1,497655] | -0,862494 |
| 130 | [0,175211, -1,497735] | -0,862494 |
| 131 | [0,174943, -1,497878] | -0,862495 |
| 132 | [0,174809, -1,497950] | -0,862495 |
| 133 | [0,175052, -1,498080] | -0,862496 |
| 134 | [0,175173, -1,498146] | -0,862496 |
| 135 | [0,174953, -1,498263] | -0,862497 |
| 136 | [0,174844, -1,498322] | -0,862497 |
| 137 | [0,175042, -1,498429] | -0,862497 |
| 138 | [0,175141, -1,498482] | -0,862497 |
| 139 | [0,174962, -1,498578] | -0,862498 |
| 140 | [0,174872, -1,498627] | -0,862498 |
| 141 | [0,175035, -1,498714] | -0,862498 |
| 142 | [0,175116, -1,498757] | -0,862498 |
| 143 | [0,174969, -1,498836] | -0,862499 |
| 144 | [0,174895, -1,498876] | -0,862499 |
| 145 | [0,175028, -1,498947] | -0,862499 |
| 146 | [0,175095, -1,498983] | -0,862499 |
| 147 | [0,174974, -1,499047] | -0,862499 |
| 148 | [0,174914, -1,499080] | -0,862499 |
| 149 | [0,175023, -1,499138] | -0,862499 |
| 150 | [0,175078, -1,499167] | -0,862499 |
| 151 | [0,174979, -1,499220] | -0,862499 |
| 152 | [0,174930, -1,499247] | -0,862499 |
| 153 | [0,175019, -1,499294] | -0,862499 |
| 154 | [0,175063, -1,499318] | -0,862499 |
| 155 | [0,174983, -1,499362] | -0,862500 |
| 156 | [0,174943, -1,499383] | -0,862500 |
| 157 | [0,175016, -1,499422] | -0,862500 |
| 158 | [0,175052, -1,499442] | -0,862500 |
| 159 | [0,174986, -1,499477] | -0,862500 |
| 160 | [0,174953, -1,499495] | -0,862500 |
| 161 | [0,175013, -1,499527] | -0,862500 |
| 162 | [0,175043, -1,499543] | -0,862500 |
| 163 | [0,174989, -1,499572] | -0,862500 |
| 164 | [0,174962, -1,499587] | -0,862500 |
| 165 | [0,175010, -1,499613] | -0,862500 |
| 166 | [0,175035, -1,499626] | -0,862500 |
| 167 | [0,174991, -1,499650] | -0,862500 |
| 168 | [0,174968, -1,499662] | -0,862500 |
| 169 | [0,175009, -1,499683] | -0,862500 |
| 170 | [0,175029, -1,499694] | -0,862500 |
| 171 | [0,174992, -1,499713] | -0,862500 |
| 172 | [0,174974, -1,499723] | -0,862500 |
| 173 | [0,175007, -1,499741] | -0,862500 |
| 174 | [0,175023, -1,499749] | -0,862500 |
| 175 | [0,174994, -1,499765] | -0,862500 |
| 176 | [0,174979, -1,499773] | -0,862500 |